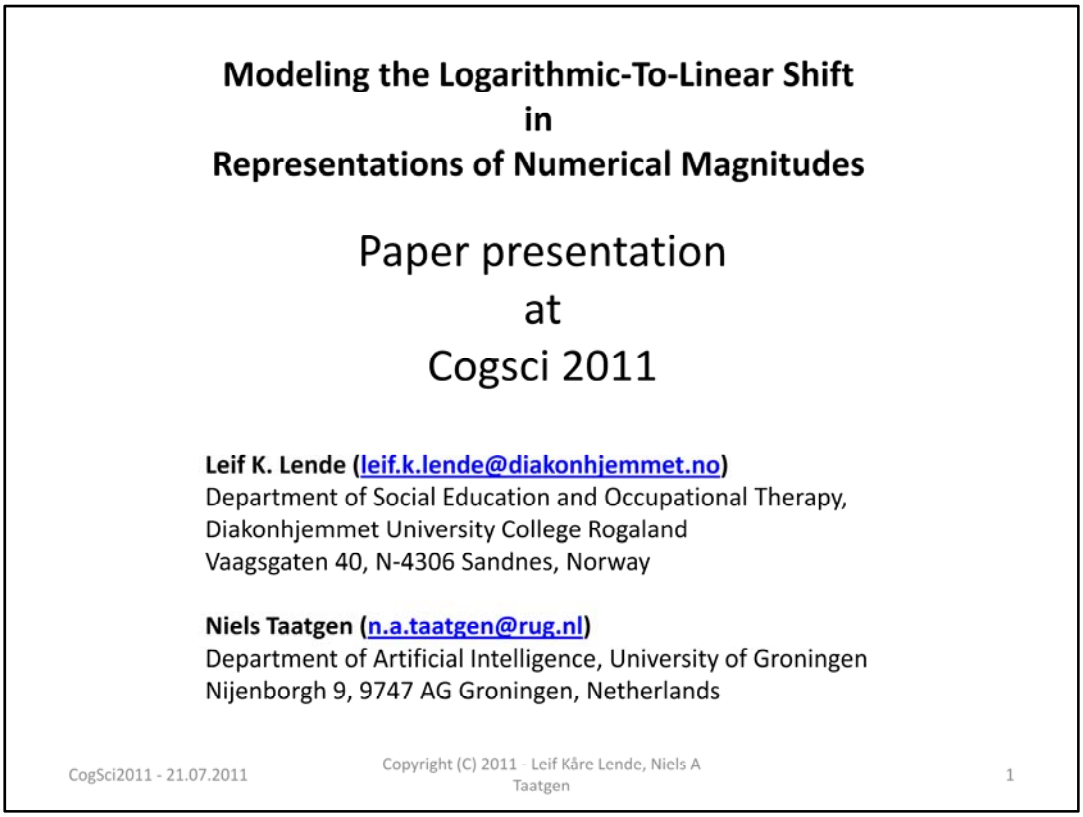
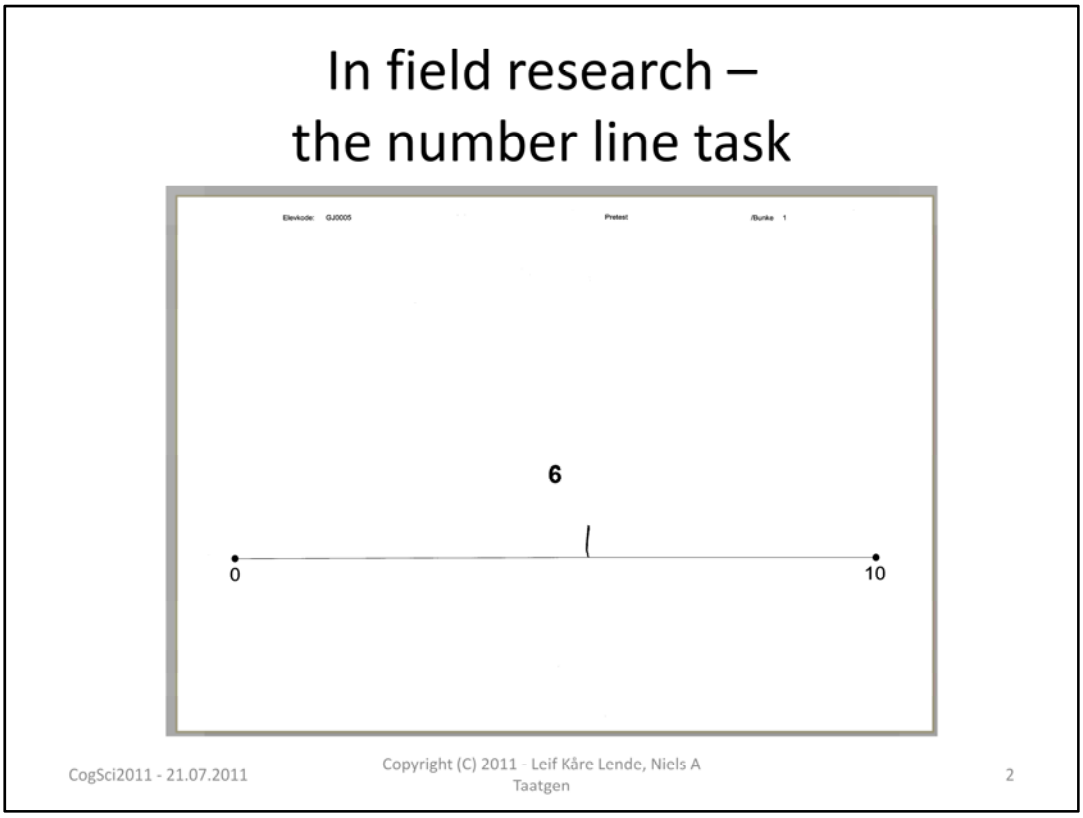


My name is Leif Kåre Lende. I'm an assistant professor at Diakonhjemmet University College in Norway, Department of Social Education and Occupational Therapy. And the second writer, professor Niels Taatgen, University of Groningen is my supervisor.

I'm presenting our paper with the title: "Modeling the Logarithmic-To-Linear Shift in Representation of Numerical Magnitudes" . The larger experiment, which the research presented in the paper is a part of, now is carried out and gives a broader and more fruitful background for our ideas of the model. For that reason I will present the whole experiment and discuss the findings relevant for our model.



One important learning process that is involved when dealing with number magnitude is the estimation of what position a number value has on a number line.

In our paper we presented our first findings from the pre-test of a larger experiment where the subjects are Norwegian 6 year old children just started on pre-school education learning numbers from 1 to 10.
In Norway the population of children at kindergarten and preschool are mixed. Thus we have defined the population for the experiment by learning level. In Norway children start at school the year they are 6 years old. This first year in school they start to learn the numbers from 1 to 10. We assume that this represents a mental level that should make most of them just capable of dealing with the empty number line.

The findings provide information on number line estimation - that is, translating a number to a spatial position on a number line. The results show different categories of representation of the magnitudes on the number line, which may represent different stages in a learning sequence. On this basis, we show a proposal for a cognitive model of the learning process towards a linear representation of magnitudes.

In this presentation we will also present correlating findings from the posttest (posttest1) and the long time learning test (posttest3). Cluster analyses of the results from all three tests supports the evidence of the four stage categories we have proposed from visual inspection of the results of the pretest.

From logarithmic to linear representation

- From Siegler, Thompson, & Opfer, (2009)

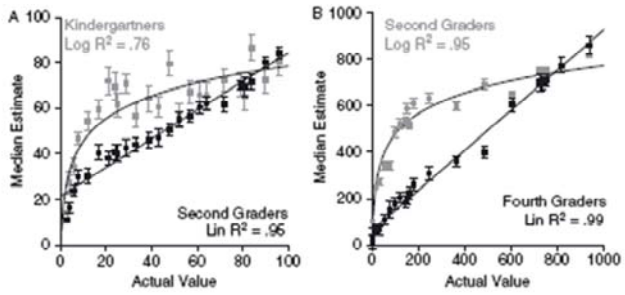
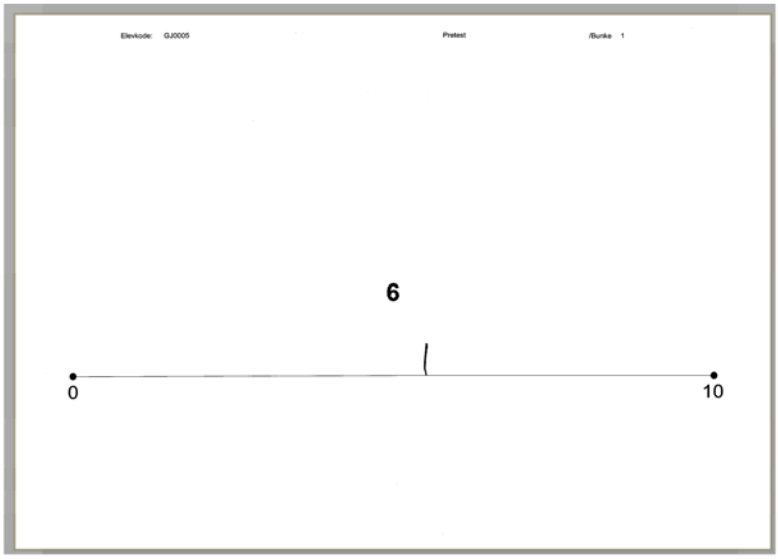


Figure 1. The logarithmic to linear shift. From Siegler, Thompson, & Opfer, (2009), Copyright 2009 Wiley. Reprinted with permission

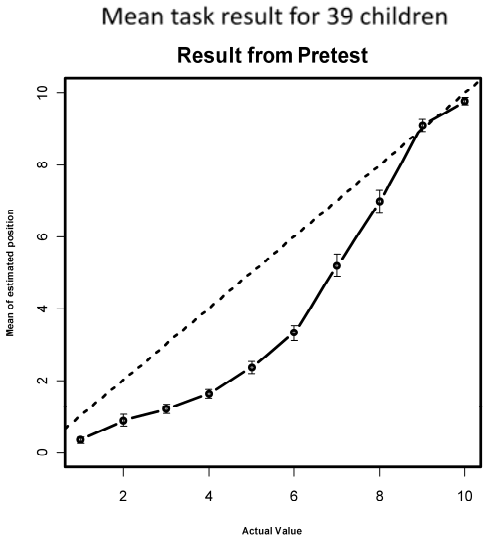
For young people it's a typical finding that they change from logarithmic to linear representation on numerosity estimation tasks. Siegler et al (2009) show that children undergo parallel changes from logarithmic to linear representation on numerosity estimation tasks. The example we have reused from their article shows long-term changes in estimation of whole number magnitudes. (A) On 0–100 number lines, kindergartners' estimates were better fit by the logarithmic function than by the linear, whereas second-graders' estimates were better fit by the linear function than by the logarithmic; (B) On 0–1000 number lines, second-graders' estimates were better fit by the logarithmic function than by the linear, whereas fourth-graders' estimates were better fit by the linear function than by the logarithmic. Siegler and Ramani (2008) made a study among preschool children from low income families which indicated that playing a simple numerical board game for four 15-minute sessions eliminated the differences in numerical estimation proficiency

Example of a test sheet from– our number line task



We did this experiment of our own . In all our tests, the stimuli for the number line estimation task were two stacks of 10 sheets of paper, each with a 25 cm long line arranged horizontally across the page, with ‘0’ just below the left end of the line, and ‘10’ just below the right end. A number from 1 to 10 inclusive was printed approximately 3 cm above the center of the line, with each number printed on one of the 10 sheets in each stack. The order of the sheets in the stack is randomized.

Our findings from the pretest



This graph shows the mapping between numbers and positions on the number line that we found in the pretest of the experiment. Performance is on average reasonably good. What is surprising is that the extent in which the curve differs from linear is not towards a logarithmic curve, but in the opposite direction.

Performance on the pretest happened to be better in the control group than in the experiment group (not shown here).

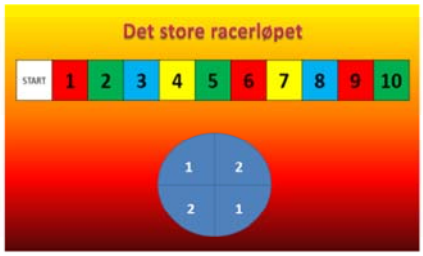
Points are plotted with error bars.

Short description of the experiment

- At each of the participating schools the subjects were randomly divided in an experiment group and a control group.
- The very beginning of the experiment was the pretest
- ..immediately followed by two weeks of training with a board game.
- Then those two weeks were immediately followed by a posttest equal to the pretest
- And nine weeks after the pretest were held: a new posttest equal to the pretest

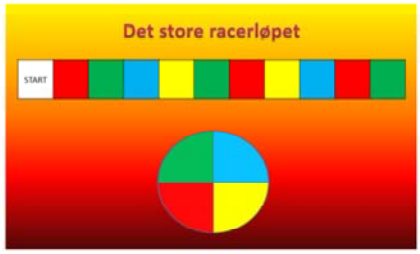
But the whole experiment contained three more elements then just the pretest:
Immediately following the pretest: two weeks of training with a board game.
Then those two weeks were immediately followed by a posttest equal to the pretest
And nine weeks after the day the pretest were held: a new posttest equal to the pretest, were held

Training with a board game



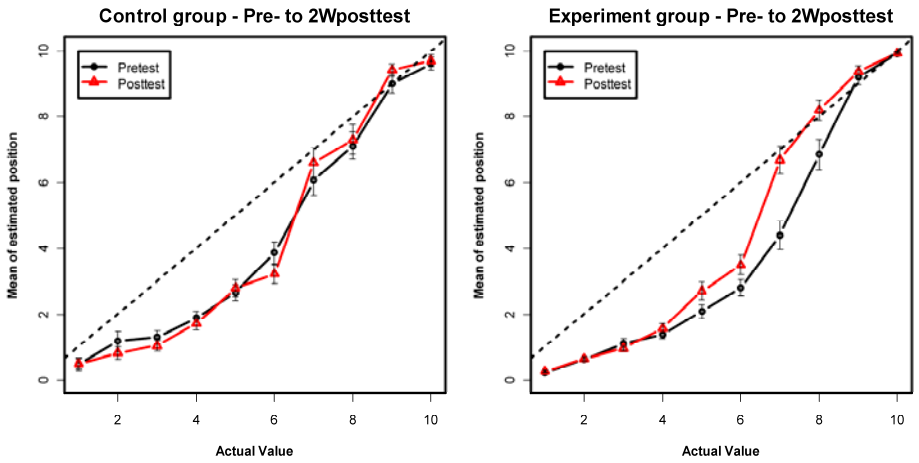
← Game board used by the experiment group

Game board used by the control group →



After the pretest, the children trained with the board game 15 minutes 2 days a week for two weeks. Then a posttest showed that the children in the experiment group made significant improvement compared to those in the control group.

Patterns pre- to 2Wposttest



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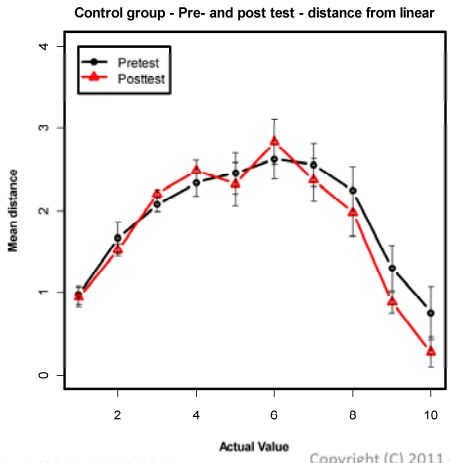
The difference in improvement between the control group and the experiment group is here shown by mean values from the result of each group's pretest and the first posttest held after the 2 weeks of training.

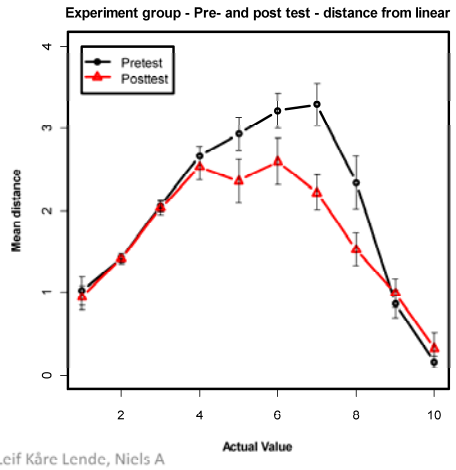
[Display sequence:]
1. First the pretest of the control group
2. Then the posttest after two weeks
We can see from this graphs of the patterns, that there are very little difference (little or no improvement)
3 Then the pretest by the experiment group
4 And the posttest.
We can see that there are a distinct difference of the two curves. The experiment group improve better than the control group.
An ANOVA-test show that the difference is significant.

Improvement from pre- to 2Wposttest

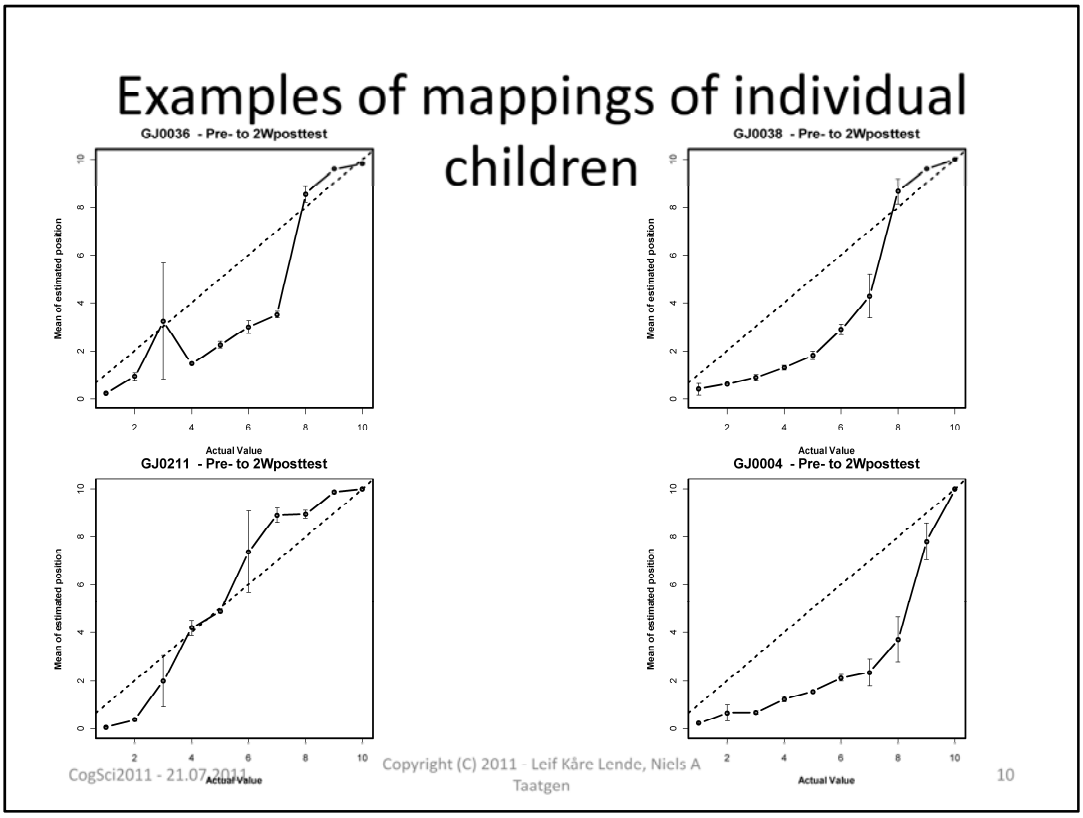
Control group



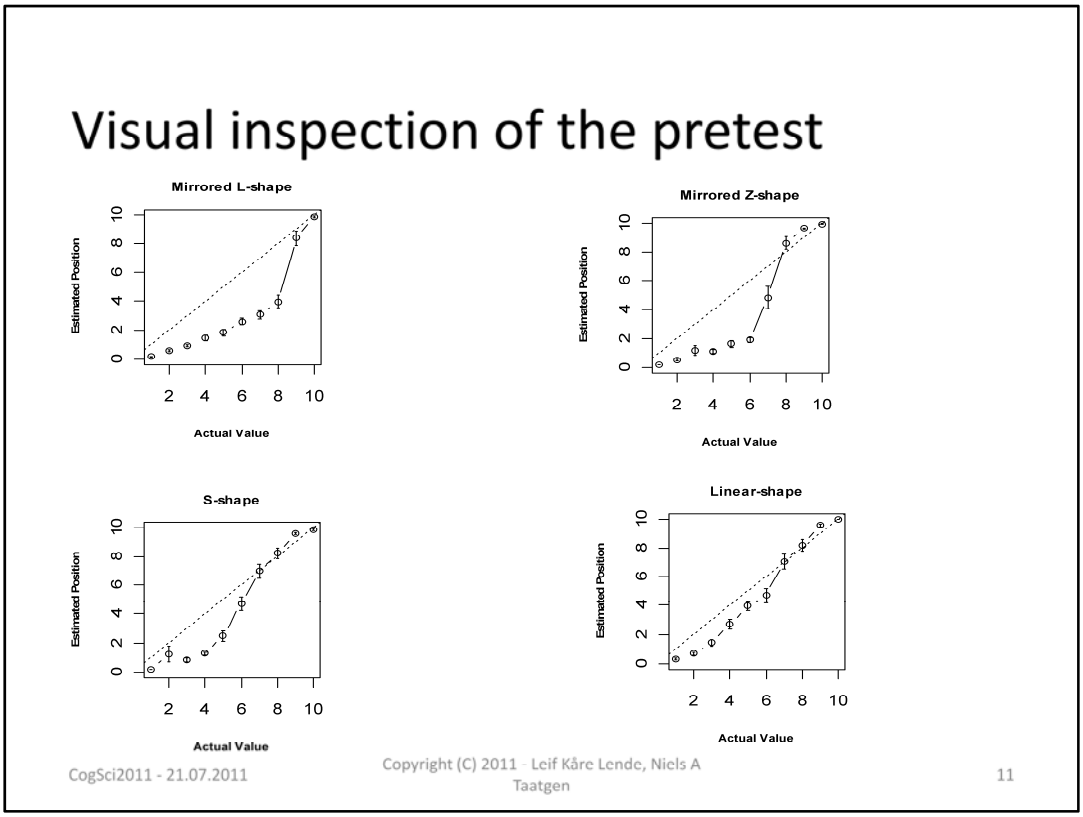
Experiment group



As we can see from these graphs too, of absolute values of mean distances from a linear representation, training with the board game improves the skills of managing the given number line task. As mentioned, we did an ANOVA on the difference between the control group and the experiment group and found that the difference is significant with most satisfying values. This adds evidence to what Siegler and Ramani (2008) has showed in earlier research.



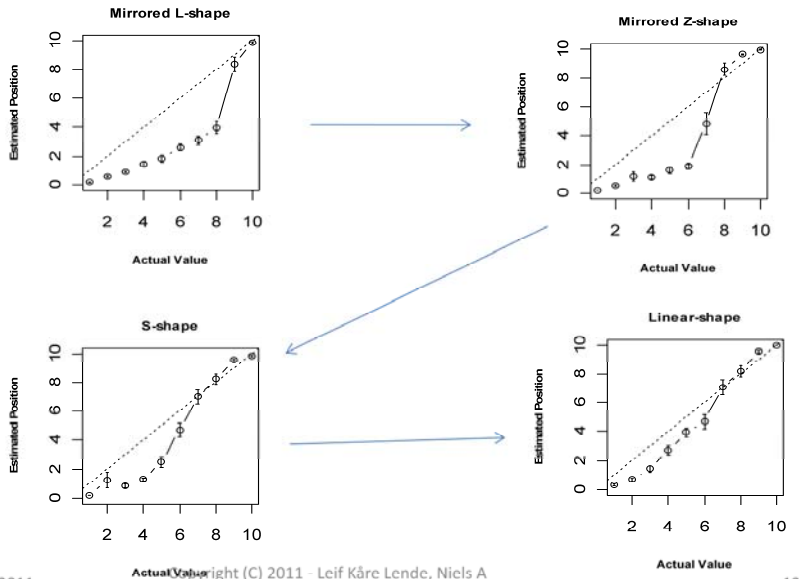
To get a better sense of the difference of responses, we already after the pretest did a visual inspection of each individual subject.
Here are some examples of mappings. It's plotted with error bars.



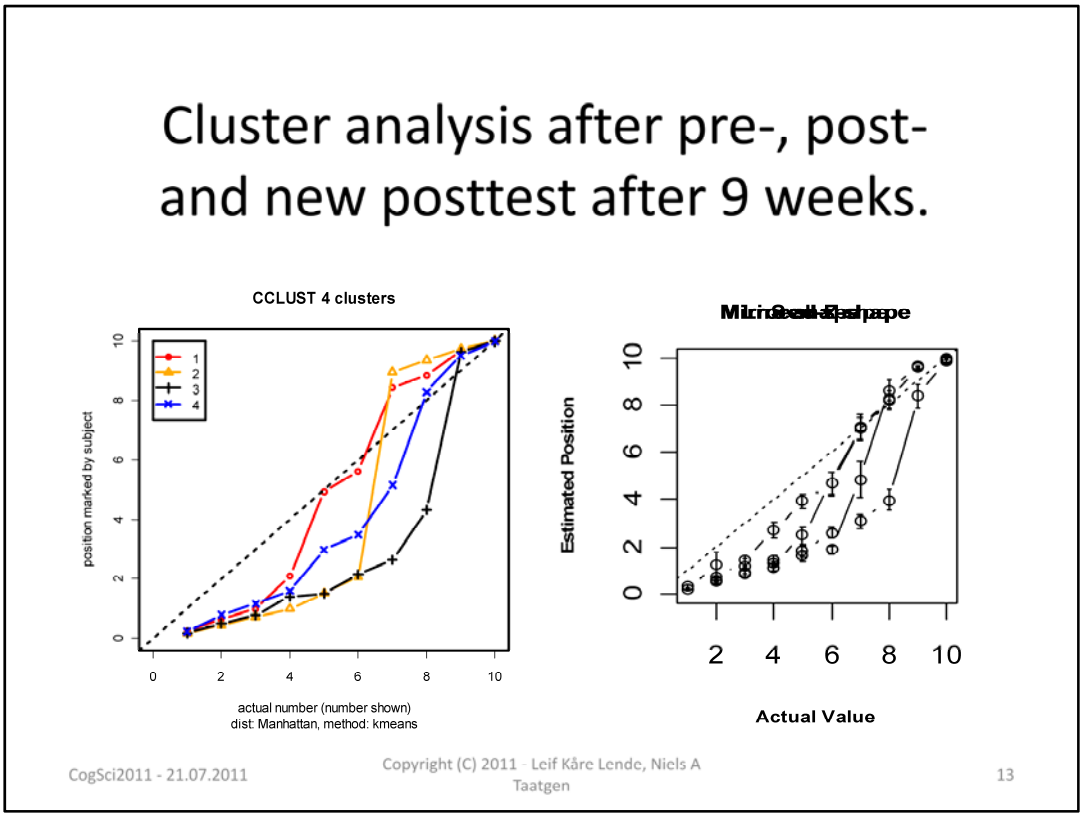
From this visual inspection, three of the participants were excluded because they produced more or less random results. We also found, that there were roughly four groups that resemble a horizontal mirrored L, a mirrored Z, a mirrored S and a linear line.
Not any of them show something like a logarithmic representation.

Hyphothesis of learning sequence

L
↓
Z
↓
S
↓
Lin



Even though the borders between these categories are fuzzy, they seem to represent a sequence in learning the number line from 0 to 10.

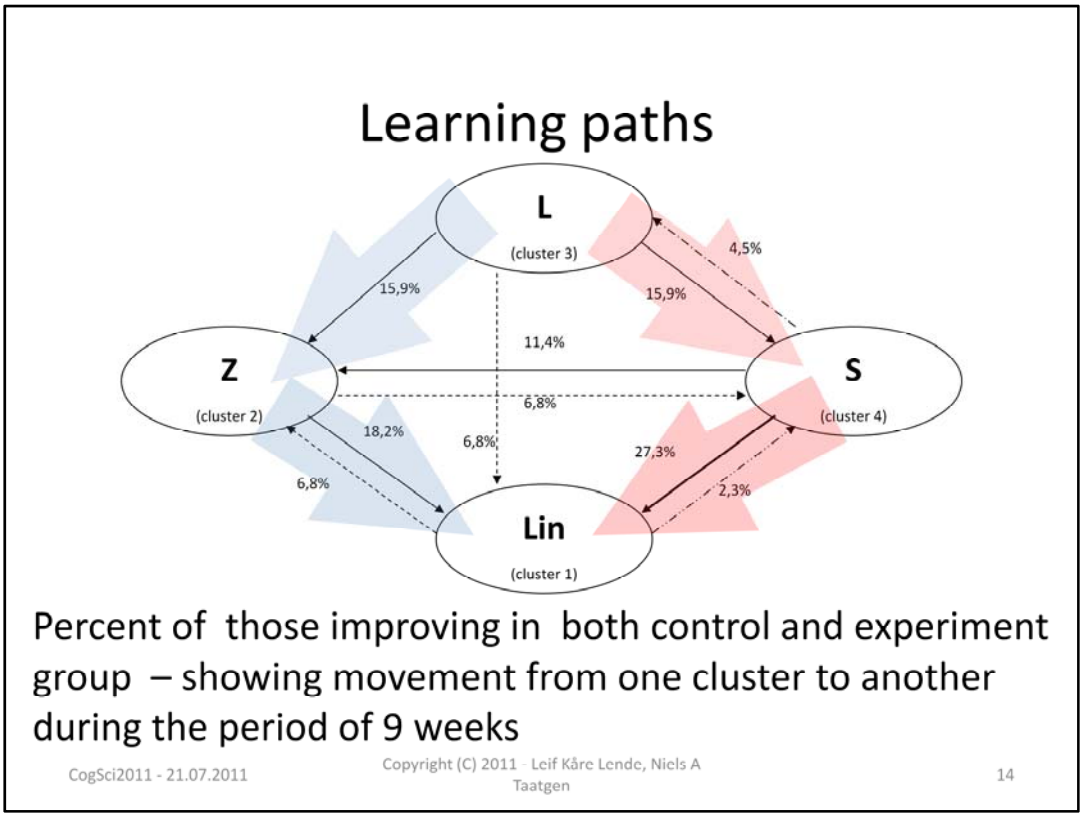


After data from two additional tests were available, the posttest after two weeks and a long time learning test after 9 weeks, we made cluster analyses of all 108 pattern records. This was done to get a better sense of the difference of responses. To categorize individuals into four categories, we used the cluster analyzing function CCLUST of CRAN-R's library CCLUST with "Manhattan", the mean absolute Error, as distance parameter and "kmeans" as clustering method.

The first one, annotated with a + in the CCLUST graph is the cluster centers of those subjects who, to a large extent, is using count up from zero as part of the strategy for solving the task. The graph of this category looks like an inverted L. Cluster 2, represented by the annotation of a Δ, is another category. The graphical representation of this category very much looks like an inverted Z. The subjects belonging to this category is considered to have learned that roughly 7,8,9 and 10 should be near 10 and they rely on counting for the positioning of all numbers. The third, one, annotated by an o, is those subjects using count up only for a bit but seems to have learned that 5 an 6 belongs more in the middle of the line and 7,8,9 and 10 should be positioned near 10 in some kind of order.
The last cluster, which is annotated x in the graph, and looks like an italic inverted S, is not that well familiar with number 5 as a mid point of the number line, but still have a better sense of which numbers are more far off from one and closer to ten.

[SHOWING THE RIGHT PART OF THE SLIDE:]
Comparing the results from the cluster analyzing of all tests with the tentative proposals in our paper, roughly shows the same graph profiles.
The L-shape graph fits with the one annotated with + in CCLUST graph
The Z-shape graph fits with the one annotated with Δ in CCLUST graph
The S-shape graph fits with the one annotated with x in CCLUST graph
The Lin-shape graph fits with the one annotated with o in CCLUST graph

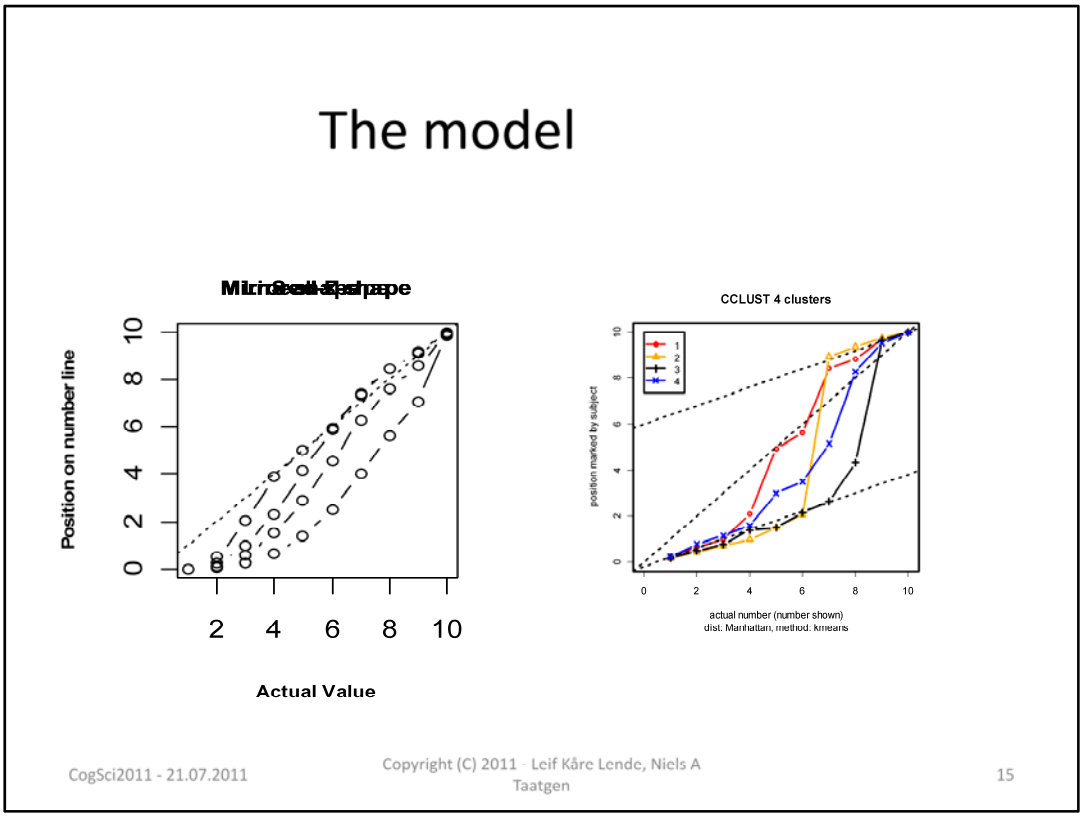
As we see, CCLUST has skewed the centers of the clusters a bit. Since our proposal from the paper was based on visual inspection of quite few data records and this cluster analyses is based on a lot more data, we must expect that the center my be skewed. For us, the profiles is the important thing, and those are basically the same, in fact visually they now are even more distinct.



We assume that one cluster relates to the least of skills , and one to the best. But how do they relate to each other?

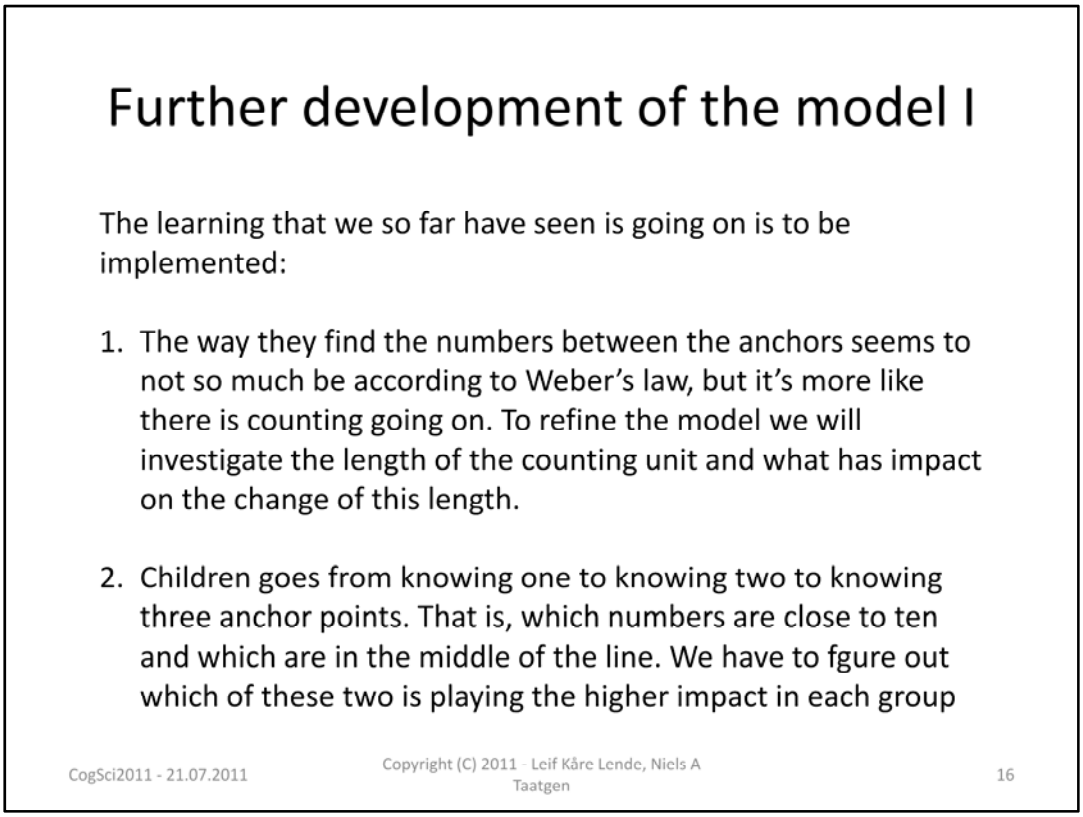
In our paper we proposed that the learning progress follows a sequential path. By looking at each individual and what steps they take from test to test, we found that there are reasons to argue against a single sequential path. We find that there are two main paths:

1. From L to S to Lin
2. and from L to Z to Lin



The model we proposed in our paper is solely based on Weber's law, anchoring and a blending function. If we compare the results given by the model, we can see that there are similarities, but what clearly lacks is the counting procedure which the subject of our in field research in several of the groups has used as part of their strategy. As we can see from the plot to the right, the mirrored Z-group and the mirrored L-group seems to use counting with rather similar length of counting steps which gives us a linear representation with the same slope for both lower and higher numbers.

In our paper we say that in the case where we get an S-shaped curve the model knows the anchor point 5 too, but with smallest activation, and 1 and 10 is strengthened with one entry each. We now clearly see that the image of the curve is more and more similar to the relevant curve image from the result of our experiment data. The reason for that, we assume, is relying on the subjects decreasing use of counting when they learn more anchor points and therefore corresponds better to our model, that as mentioned in our paper, do not use counting but only blending.



The learning that we so far have seen is going on is to be implemented:

1. The way they find the numbers between the anchors seems to not so much be according to Weber's law, but it's more like there is counting going on. To refine the model we will investigate the length of the counting unit and what has impact on the change of this length.

2. Children goes from knowing one to knowing two to knowing three anchor points. That is, which numbers are close to ten and which are in the middle of the line. We have to fgure out which of these two is playing the higher impact in each group

Further development of the model II

3. But there might even be a third strategy involved which Barth & Palladino (2011) calls proportioning, and this one is especially represented in the S- and Lin-shape group. We have to figure out how proportioning and calibrating of the counting unit seems to be done. Is it only by division, or is it also done by try and fail.

When an individual has finished learning to do the number line task perfect, we could say that he/she has figure out the right proportion of the counting unit. Most of the children we tested don't do that. So even if they to one extent do that, very few of them manage to do the number line task perfect.

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Siegler, R. S., & Ramani, G. B. (2008). Playing linear numerical board games promotes low-income children's numerical development. *Developmental science*, *11*(5), 655-61.

For references used in the paper, see reference list of the paper.

Thank you for listening

Further questions may be addressed to:

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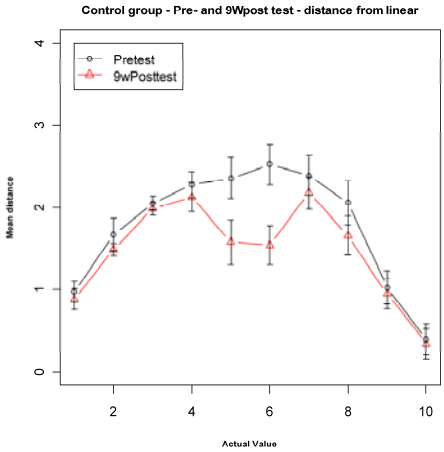
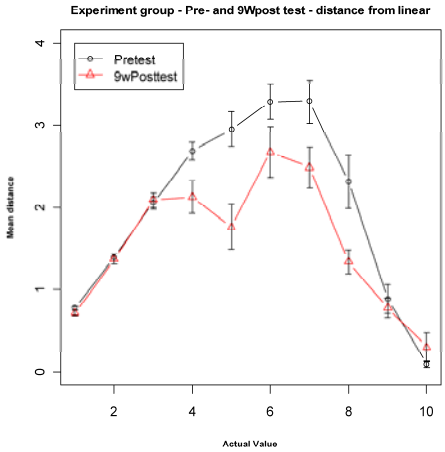
http://hjem.diakonhjemmeths.no/lende

Improvement from pre- to 9Wposttest

Control group



Experiment group

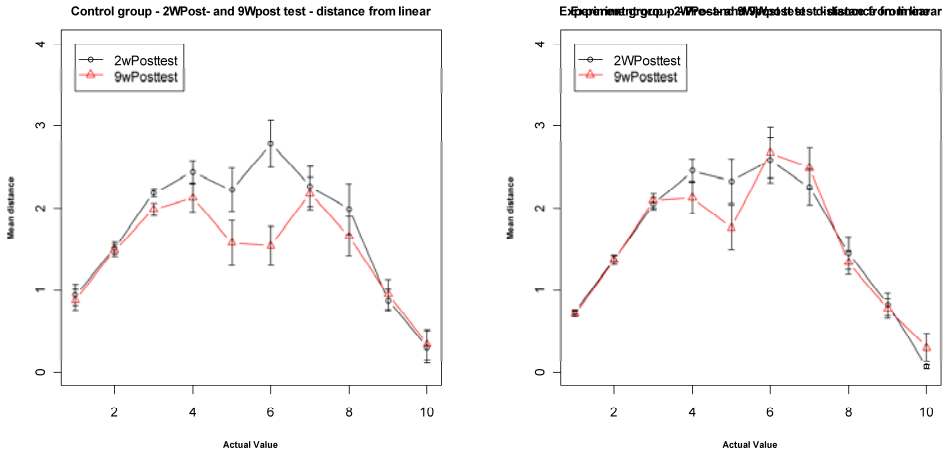


Spared if needed for answering questions

Improvement from 2Wposttest- to 9Wposttest

Control group

Experiment group



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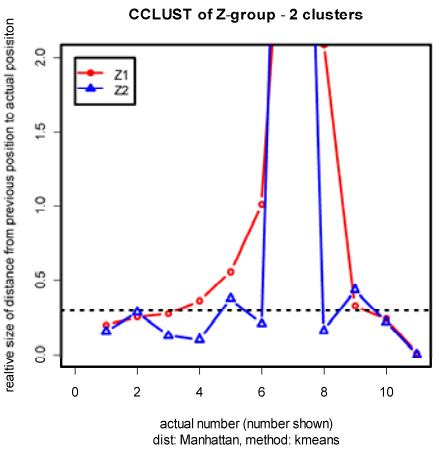
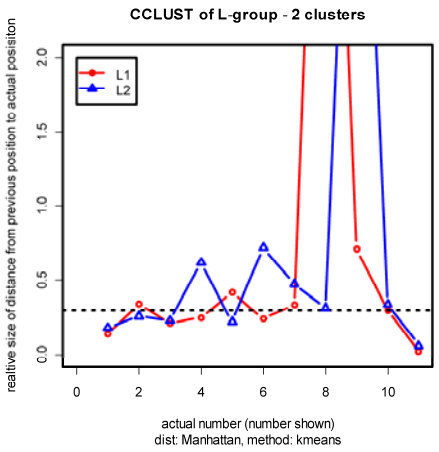
The count-unit





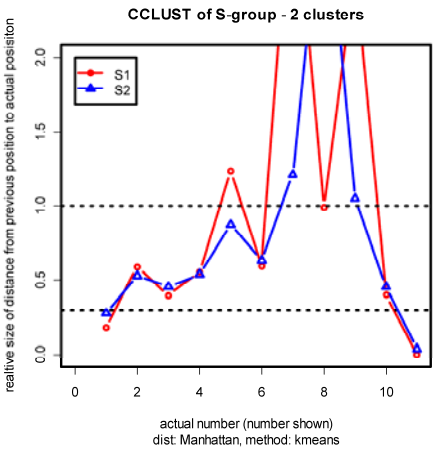
Spared if needed for answering questions

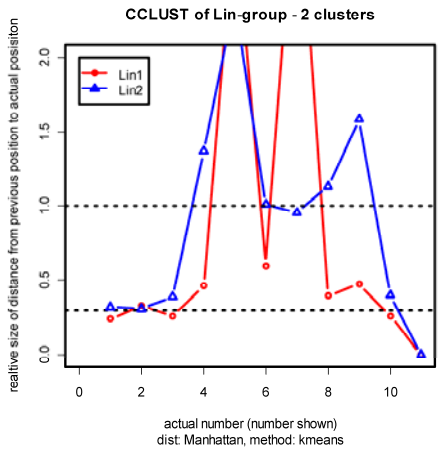
By investigating the individuals using the strategy found for the mirrored L-group and the mirrored Z-group, which almost solely seems to rely on counting, either up from zero or down from 10, we find that the length of the counting unit is rather constant and for an average a lot smaller than one. The average is about 0.3 of the actual unit of length between to numbers on the number line, which on the sheets that we used for the number line task is about 7.5 mm. We don't know if this would be the same for an individual in the Z- and L-groups if the number line were of another length, double length or half length. That's why there is a need of a new experiment with the same number line task but with empty number lines of different length.

Which of these two is playing the higher impact in each group

To different new experiment
Computer dragging mous
Different lenght of lines

Proportioning and calibrating





Spared if needed for answering questions

A closer look at the S- and Lin-groups show that the individuals has better proportions for several of the units of length between subsequent numbers. But we can also see that the anchor point 5 plays a role, especially for the Lin-group.